

Definition of i and Operations with Complex Numbers

1. Definition of i

The imaginary unit, denoted by i , is defined as the square root of -1 :

$$i = \sqrt{-1}$$

From this definition, the following powers of i are fundamental:

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1 \text{ (and the pattern repeats every 4 powers).}$$

2. Complex Numbers

A complex number is a number of the form:

$$z = a + bi$$

where a is the real part and b is the imaginary part of the real number.

3. Operations with Complex Numbers

a) Addition and Subtraction

Add or subtract the real parts and the imaginary parts separately:

$$(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$$

b) Multiplication

Use the distributive property (FOIL method) and apply $i^2 = -1$:

$$(a + bi)(c + di) = ac + adi + bci + bdi^2$$

c) Division

Multiply numerator and denominator by the conjugate of the denominator:

$$\frac{(a + bi)}{(c + di)} \times \frac{(c - di)}{(c - di)}$$

Notice that: The conjugate of $z = a + bi$ is $\bar{z} = a - bi$.

It is used to simplify division and has the property:

$$z \times \bar{z} = a^2 + b^2$$

<https://mystatclass.com/mac1105w.html>